Slide Rules and Logarithm Tables

MATHEMATICAL
INTENTIONS
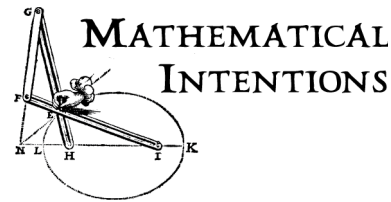
A. Learn to use a slide rule.

Try this (1).

a) Obtain or make a slide rule. Do one of the following.

   i) Use the SlideRule.html online, or do a web search and find another virtual slide rule OR

   ii) Print out PaperSlideRule.pdf, and cut and assemble it OR

   iii) Make a cardboard slide rule following the instructions at http://www.sphere.bc.ca/test/build.html OR

   iv) Find an actual slide rule. (Anyone who studied science before 1970 probably used a slide rule, and may have one you can try.)

b) Learn to multiply with the slide rule. Start with these examples, then try some more of your own.

Example 1.   2x3

   a. Use the C and D scales on the slide rule. (A and B scales will also work.)

   b. Move the C rule so that the leftmost 1 is aligned with 2 on the D rule.

   c. Move the hairline cursor so that it is aligned with 3 on the C rule.

   d. Read down along the cursor to the D rule to find the product, 6.
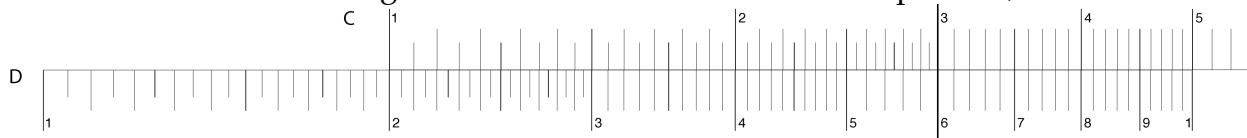
Example 2.   4x9

   a. If you move the C rule so that the leftmost 1 is aligned with 4 on the D rule, the 9 will not be over the D rule.
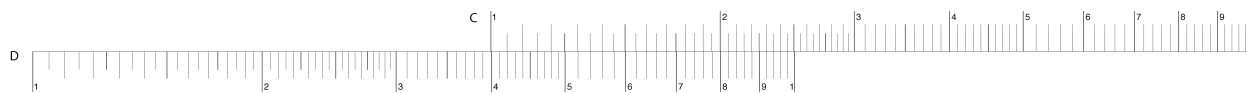
   So, instead, move the C rule so that the *rightmost* 1 is aligned with 4 on the D rule.

   b. Move the hairline cursor so that it is aligned with 9 on the C rule.

   c. Read down along the cursor to the D rule to find the product. It ought to be 36, but it's between 3 and 4. Actually, it's at 3.6. On a slide rule, you have to figure out the place values mentally; the result of your computation may have to be multiplied by a power of 10.
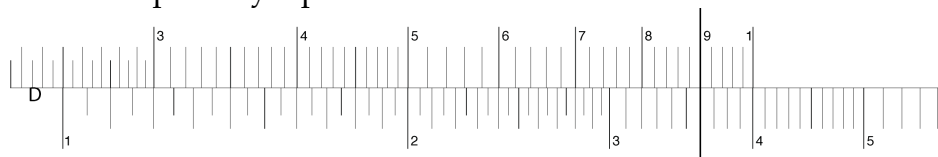
Example 3.   52 x .0007

a. Use the slide rule to find 52 x .0007. The product appears to be about 3.5.
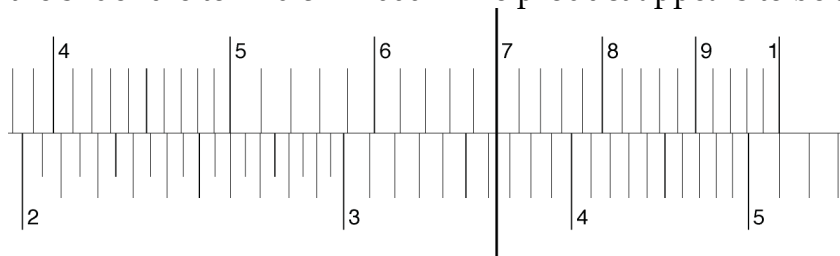
b. Use scientific notation to figure out the place values:

$$52 \times .0007 = (5.2 \times 10) \times (7 \times 10^{-4})$$
$$\approx (5 \times 7) \times (10^{1} \times 10^{-4})$$
$$= 35 \times 10^{-3}$$
$$= .035$$

c. So, looking at the scales more carefully, the product is close to .0363. If you had multiplied it out by hand, the rightmost digit would be 4 (from 2x7), so the exact product is .0364.

c) Think about how the numbers are spaced on the rules.

   i) Measure the spacing.

      (1) Use a physical slide rule, and a piece of paper to measure with, or use SlideRule2.html and use the Measuring stick segment. If you are using the PaperSlideRule.pdf, print up more copies of the D scale and tape one onto the left end and one onto the right end, leaving no gaps.

      (2) Mark the length from 1 to 2 on the D scale. Move the paper or measuring stick (in the applet, grab it in the middle) to start at 2, and measure off an equal distance. What number is marked there on scale D? Repeat, generating a sequence of numbers that are physically spaced equally on the rule. What kind of sequence is this?
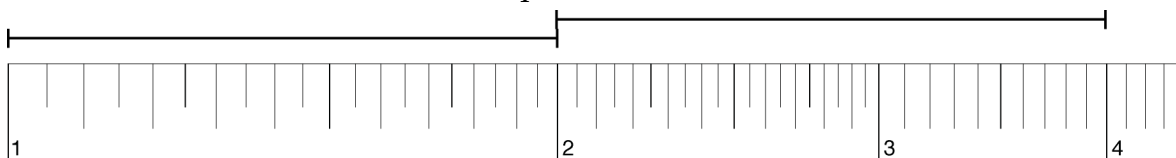
      (3) If you are using the applet SlideRule2, there is a second copy of scale D that starts at the right end, and another at the left. To make the sequence make sense, the copy to the right should be in 10s, and the copy to the left should be in tenths.
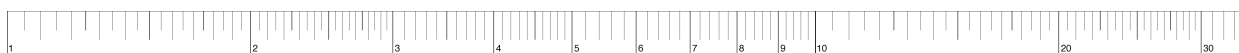
      (4) Repeat, starting with the distance from 1 to 3. Do some others, too, including 10.

   ii) When you multiply with a slide rule, you are really adding distances. However, the distances are NOT the numbers marked on the rule. How are the marked numbers related to the actual distances? How is this related to the rules for exponents and logarithms? See SlideRuleAdd.html to really add distances. See CircularSlideRule.html for an idea of how to deal with the fact that everything repeats on the second copy.

d) Accuracy. Clearly there is a limit to the number of digits you can accurately read off a slide rule.

   i) How many significant figures can you read off the left end of your slide rule?

   ii) How many significant figures can you read off the right end of your slide rule?

   iii) If you multiply two numbers with 3 digits of precision, how many digits can you read off for the product?

e) Figure out how to divide with the slide rule.

f) Use the A and D scales to find squares and square roots of numbers.

   i) If you are using the online slide rule, the left edges are aligned; drag point A down to compare the scales more easily. If you are using a physical slide rule, align the left edges of the A and D scales and use the cursor to see what numbers correspond.
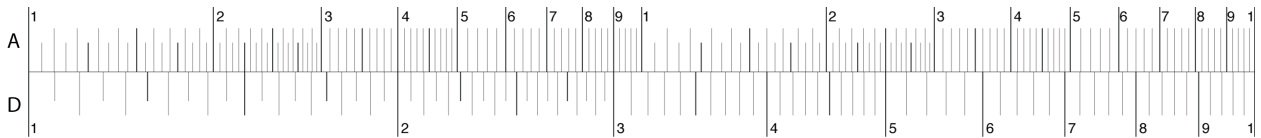
   ii) Explain how and why this works using your explanation for part c)ii).

B.   Make a logarithm table

Historically, logarithm tables were created by coordinating an arithmetic sequence (a sequence generated by repeatedly adding the same number) with a geometric sequence (a sequence generated by repeatedly multiplying by the same number), then filling in the gaps in a way that preserves the adding and multiplying patterns.

Here's an example using the arithmetic sequence $0, 1, 2, 3, \ldots$ (generated by adding 1) and the geometric sequence $1, 2, 4, 8, \ldots$ (generated by multiplying by 2.)

Table 1

The hard part is figuring out how to interpolate (fill in the gaps) in such a way that the arithmetic column remains a constant addition and the geometric column a constant multiplication. In the table below, we are splitting the gaps in the arithmetic

sequence into two equal parts. What should the constant multiplier be for the split geometric sequence?

Table 2

arithmetic sequence	geometric sequence	
0	1	
+0.5		×?
0.5		
+0.5		×?
1	2	
+0.5		×?
1.5		
+0.5		×?
2	4	

It should take two equal multiplication steps to get from 1 to 2. But multiplication by what? Use a little algebra:

$$1 \times ? \times ? = 2$$

$$\text{so} \qquad ?^2 = 2$$

$$\text{so} \qquad ? = \sqrt{2}$$

Table 3

arithmetic sequence	geometric sequence	
0	1	
+0.5		×$\sqrt{2}$
0.5	1.4142	
+0.5		×$\sqrt{2}$
1	2	
+0.5		×$\sqrt{2}$
1.5	2.8284	
+0.5		×$\sqrt{2}$
2	4	

Historically, efficient algorithms existed for finding square roots (see SquareRoots.html), so this calculation could be done fairly easily.

Here (Table 4) is the x2 table, interpolated by splitting in half 5 times. The multiplicative splitting was done with the square root function in a spreadsheet. Values in the geometric sequence column have been rounded to the 9th decimal place.

Table 4

arith. (+1)	geom. (x2)	arith. (+1)	geom. (x2)	arith. (+1)	geom. (x2)
0	1	1	2	2	4
0.03125	1.021897149	1.03125	2.043794297	2.03125	4.087588595
0.0625	1.044273782	1.0625	2.088547565	2.0625	4.17709513
0.09375	1.067140401	1.09375	2.134280801	2.09375	4.268561603
0.125	1.090507733	1.125	2.181015465	2.125	4.362030931
0.15625	1.114386743	1.15625	2.228773485	2.15625	4.45754697
0.1875	1.138788635	1.1875	2.27757727	2.1875	4.555154539
0.21875	1.163724859	1.21875	2.327449718	2.21875	4.654899435
0.25	1.189207115	1.25	2.37841423	2.25	4.75682846
0.28125	1.21524736	1.28125	2.43049472	2.28125	4.86098944
0.3125	1.241857812	1.3125	2.483715624	2.3125	4.967431248
0.34375	1.269050957	1.34375	2.538101914	2.34375	5.076203829
0.375	1.296839555	1.375	2.593679109	2.375	5.187358219
0.40625	1.325236643	1.40625	2.650473286	2.40625	5.300946573
0.4375	1.354255547	1.4375	2.708511094	2.4375	5.417022188
0.46875	1.383909882	1.46875	2.767819764	2.46875	5.535639528
0.5	1.414213562	1.5	2.828427125	2.5	5.656854249
0.53125	1.445180807	1.53125	2.890361614	2.53125	5.780723228
0.5625	1.476826146	1.5625	2.953652292	2.5625	5.907304584
0.59375	1.509164428	1.59375	3.018328855	2.59375	6.03665771
0.625	1.542210825	1.625	3.084421651	2.625	6.168843302
0.65625	1.575980845	1.65625	3.15196169	2.65625	6.30392338
0.6875	1.610490332	1.6875	3.220980664	2.6875	6.441961328
0.71875	1.645755478	1.71875	3.291510956	2.71875	6.583021913
0.75	1.681792831	1.75	3.363585661	2.75	6.727171322
0.78125	1.718619298	1.78125	3.437238596	2.78125	6.874477192
0.8125	1.75625216	1.8125	3.512504321	2.8125	7.025008641
0.84375	1.794709075	1.84375	3.58941815	2.84375	7.1788363
0.875	1.834008086	1.875	3.668016173	2.875	7.336032346
0.90625	1.874167634	1.90625	3.748335268	2.90625	7.496670536
0.9375	1.915206561	1.9375	3.830413123	2.9375	7.660826246
0.96875	1.957144124	1.96875	3.914288248	2.96875	7.828576497

Multiplying by adding

Table 4 was creating by coordinating adding in the left column with multiplying in the right column. This gives a method for multiplying by using the table and adding.

Try this (2). Use versions of the +1/x2 table to multiply by adding.

a) First check it out on the version in Table 5, which has positive and negative integers in the arithmetic column.

Example 4. 4x8.

    a. Find 4 and 8 in the geometric column.

    b. Read left to find the corresponding entries in the arithmetic column.

    c. Add these entries, and find the sum in the arithmetic column.

    d. Read right to find the product of 4 and 8.

b) Multiply some more combinations of numbers in the right column of this table.
c) How is this related to the laws of exponents/logarithms?
d) Figure out how to divide numbers in the right column using the left column.
e) Figure out how to raise numbers in the right column to integer exponents using the left column.

Table 5

-4	0.0625
-3	0.125
-2	0.25
-1	0.5
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	256

f) Now try multiplying some decimals using Table 4 on the previous page, such as 1.38 by 1.82. You will not get an exact answer. Why?

Try this (3).

a) Using only addition, subtraction, multiplication, division, and the square root function, explain how to make a table of base **ten** logarithms accurate to three decimal places. Then do the calculations and make the table.

Suggestions:

- Use a spreadsheet, such as Excel, OpenOffice Calc, Google Docs, or Apple iWork Numbers. GeoGebra also has a spreadsheet window (open it with the

View menu); it has fewer spreadsheet features, but integrates well with any geometric objects you may want to construct.
- If you would like to do it more by hand, use a calculator (a 5-function calculator such as the kind that you get at a dollar store, or the one on your cell phone, is all you need. The 5 functions are $+$, $-$, x, $\div$, and $\sqrt{}$.)
- If you *really* want to do it by hand, use on of the methods in SquareRoots.html.

b) Show example calculations to explain exactly how your system works and how you know when to stop. Include in your examples some very large and some very small numbers. Check your work with a calculator.

c) Show how to use this table to approximate cube roots or fifth roots of any number.

d) Show how to use the table you made to mark off the C-D scales on a slide rule.

Try this (4).   For further exploration

a) Suppose you wanted 4 decimal places of accuracy. What more would you have to do? How much additional work would 7 places require? 10 places? When you increase the precision by 1 digit, how many times as much space would it take for the more accurate tables?

b) Go to a library and find a book of logarithmic and trigonometric tables. (For instance, (Vega, 1948) is a reprint of a book of tables from the 1700s. It gives 7 figures of accuracy. The logarithms of numbers from 1 through 10,000, in increments of 0.1, take up the first 185 pages of the book, in small type.) Read the introduction and learn how to use the tables for calculation.

References

Vega, G. F. V. (1948). *Seven place logarithmic tables of numbers and trigonometrical functions*. New York: Hafner Publishing Co.