The Role of Historical Studies in Mathematics and Science Educational Research

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Introduction

Some of the most profound educational research in mathematics and science has employed historical studies of the origins of mathematical and scientific concepts. These studies have proven to be fruitful in the design of curricula, in the creation of environments for teaching experiments, and in the formation of theories of cognition. Piaget and Vygotsky both espoused forms of "genetic epistemology," which compel educators to examine the historical, social, and cultural genesis of all knowledge (Confrey, 1994b). Differences in theoretical framework and methodology direct educational researchers to study and use historical materials differently, and this chapter will address such questions as: What kinds of historical investigations are desirable? Where and how should they be presented and discussed? What sort of reforms of curricula can history inspire? What kind of history, if any, should be presented directly to secondary students? or to teacher candidates? What part should history play in educational philosophy and epistemology?

Responses to these questions will be framed in three different approaches to the use of the history of mathematics and science in educational research. These three approaches to historical and educational research require increasing levels of scholarly engagement with historical materials and offer correspondingly increased levels of insight. Each of these approaches is discussed in the three subsequent sections of this chapter. Summarized briefly, they are:

- Historical background as an addendum to traditional curricula, used mainly to inspire students and to lend to mathematics and science a humanistic face and to give an idea of their place in culture.

- The use of original, historical, source material to gain insights into the problems, situations, and intellectual environments that led to the genesis of scientific concepts, focusing on alternative and diverse views that no longer exist in standard modern curricula but could be revised and revived in light of new educational situations.

- Study of the broader social history in which the original sources of mathematics and science are embedded in order to see how certain views came to be valued over others and subsequently enshrined in traditional curricula; that is, the history of the values implied by our choice of curricula.

These three directions for historical and educational research can be summarized as: context, content, and critique. In other words, the first approach provides students with a cultural context for existing curricula. The second approach provides researchers with new ideas for curricula and for the design of learning environments. The third provides researchers with the tools to engage in a broad critique of existing curricular concepts in order to redirect education in the service of the larger society and its newly emerging goals.

Educational research is inherently interdisciplinary and, therefore, can place exorbitant intellectual demands on researchers. The researcher must combine an expertise on student voice and perspective with a detailed knowledge of mathematical and scientific concepts. This already requires a researcher to wear two hats. In addition, I am proposing that such researchers could benefit greatly from knowledge about the historical origins of science itself, as evidenced in the documents that reveal the voice and perspective of its genesis. I am aware that this seems to place unrealistic demands on any one researcher (three hats? four hats?), but a successful educational research program is nearly always a cooperative endeavor, and it seems quite realistic that such programs could include a historian on the team or at least the possibility to consult with one occasionally, although, in order to be useful, such a historian would have to be acutely aware of the direction of the research and be able to locate and interpret appropriate historical material.

History and its uses have little pretense to objectivity; someone tells a story and such stories are rooted deeply in particular perspectives. History as an educational research methodology must begin with the establishment of an appropriate stance from which to conduct historical research. Hence, before laying out a methodological theory that integrates education, history, mathematics, and science, I will begin with a brief description of how I came to be engaged in such activity. My initial training was in mathematics, but as I began to attempt a doctoral thesis I became intensely curious about the history of mathematics, so much so that I abandoned my first thesis and began reading original source material. The original works of Gauss, Galois, Lagrange, Euler, Leibniz, Newton, Pascal, Descartes, or Apollonius were not at all what I would have expected from my modern studies based on modern textbooks. I also found that even the best modern mathematical training left me sadly lacking in a variety of backgrounds that were fundamental to understanding historical development; for example, geometry, physics, engineering, and technology, to name a few. I also found that there existed scant opportunity to write a doctoral dissertation on the history of mathematics, so I pursued my interests privately for over a decade while teaching at small colleges as a mathematics instructor.

My introduction to educational research began when I gave up teaching for a few years and went to work as a research assistant for Jere Confrey's Mathematics

Education Research Project, which was funded by the National Science Foundation at Cornell University. I studied various theories of intellectual development and research methodologies, focusing first on radical constructivism and videotaped clinical teaching interviews. I wore three hats: those of mathematician, historian, and educational researcher. In Dennis (1995) I integrated all three of these areas of research, and I continue to see huge opportunities in this direction (these would correspond roughly to the second approach listed above). I have begun to gain enough background in social history only recently to effectively pursue the third. Therefore, my fourth hat as a social historian is being woven still; the demands are great but the possible benefits are tremendous. For these reasons, in the following sections the illustrative sketches will all be taken from the history of mathematics, physics, engineering, and technology.

Each of the three approaches listed above stems from quite different theoretical imperatives. The first approach is fairly consistent with a progressive absolutist view of science. It can be used to enhance traditional curricula without making broad changes; however, when done carefully, some direct content and pedagogical changes are inevitable. The second approach emerges from the works of Piaget and the radical constructivists in that it calls for the design of innovative curricula and learning environments that are informed by genetic epistemology (Piaget & Garcia, 1989). However, within this approach, historical genesis is taken as mostly the history of ideas and concepts in and of themselves largely divorced from a broader social context. The third approach is tied more to a Vygotskian framework where the genesis of knowledge must be seen as a socially mediated construction (Wertsch, 1985). All of these approaches have certain benefits and they are not mutually exclusive in terms of classroom practice, even though it might be difficult or impossible to reconcile the differences in theoretical framework (Confrey, 1994b).

Each of the following three sections will include brief examples of some historical material and their possible implications for the reform of curricula. In order to illuminate the three different historical approaches, the examples will focus mostly on the same subject: analytic geometry. Although this subject no longer exists as a full course in high school or college, analytic geometry nevertheless remains one of the fundamental topics in secondary school mathematics and as a part of calculus, constitutes a stumbling block that impedes the progress of students in all of the sciences. A great many curricular changes are occurring in the reform of analytic geometry and calculus without enough direct discussion. Some of these changes are related to changing technology, such as graphing calculators and computers. As we shall see, the study of history can be a great aid in rethinking these curricula especially when it is combined with other educational research methods.

Historical Background as an Addendum to Traditional Curricula

Jahnke (1994) argues for the importance of seeing mathematics in a cultural setting. In order to accomplish this, he suggests that secondary school students be exposed to historical material and possibly some original historical sources that complement and diversify conceptually the traditional curricula. Mathematics entirely stripped of its origins and cultural setting is called by Jahnke "fast food mathematics," and, he argues, it cannot be appreciated fully nor comprehended entirely. He advocates exposing students to the questions and problems that led to the genesis of mathematics, and he sees this exposure as an important, but separate, part of curricula that does not fundamentally change content or pedagogy. Jahnke reminds us that "history of mathematics is difficult!" (1994, p. 141), and he feels that teachers with limited classroom time cannot fail to discuss the usual content material.

A mathematician and historian like Jahnke is suggesting the use of carefully selected and profound historical material that will illuminate and situate important concepts culturally, but a word of caution is necessary. Recently, many mathematics and science textbooks have begun to include brief historical pieces, usually biographical, even though many of these tidbits are oversimplified, misleading, or incorrect. For example, several textbooks have a picture and brief biography of René Descartes (1596-1650) at the beginning of a unit on graphing linear and quadratic equations. Descartes never used equations to plot points and create curves, and a scholar like Jahnke would never make such a blunder. Such attempts at history serve only to perpetuate the mythological status of a few selected scientists, often to the detriment of students for whom the subject is mystified further.

So what would an appropriate historical addendum for our example, analytic geometry, look like? It would have to begin with a discussion of François Viète (1540-1603) and the evolution of "syncopated algebra," that is, the symbolic algebra that the students are learning, and then go on to discuss the movement towards using such language as a possible model for geometry. During the early seventeenth century, within this movement, Descartes and Pierre de Fermat (1601-1665) approached this problem independently and simultaneously. Descartes started with mechanical devices that drew curves, then studied the motion of such devices, and found ways to represent and classify them with coordinates and equations. In the opposite direction, Fermat studied equations and how they could be represented geometrically by the curves traced by the ends of line segments with appropriately variable lengths (i.e., graphs). Descartes demonstrated, for example, that no matter how you draw a conic section, and no matter what coordinate system you choose, the resulting equation always will have degree two. On the other hand, Fermat demonstrated that given any equation of degree two, no matter how you represent it with line segments, the resulting locus always will be a conic section. Descartes and Fermat both made free use of coordinate systems with arbitrary angles between the axes. Descartes went on in his work to try to represent mechanics and physics mathematically, while Fermat became more concerned with patterns in tables of numbers, maximum-minimum problems, and number theory.

This brief synopsis is over-simplified, but an important historical point could be made to students. This is that there are two distinct approaches to analytic geometry: one that begins with mechanically or geometrically constructed curves and then attempts to represent them in the language of algebraic equations, and the other that begins with data or equations and then plots a graph as a representation in order to gain insight into the nature of numerical phenomena (e.g., maximum values). If there is to be no fundamental change in the curricula it should be pointed out to students that they will be dealing almost entirely with the latter activity, namely the approach of Fermat.

At this point, Jahnke probably would advocate showing students at least a brief example of a problem taken directly from Descartes' *Geometry* (1638) and one from Fermat (Mahoney, 1973). Then a relevant question would be where might an educational researcher get hold of appropriate historical material at this level. Although the sources cited above generally are found in university libraries they can be difficult for a non-mathematically trained historian to read. Lately, however, such mathematical history has become much more widely accessible in more reliable secondary sources such as Katz (1993), or in annotated source books such as Callinger (1995) or Struik (1969), which make selected original material much more readable. Similar publications have come out in all of the sciences (e.g., Hagen, Allchin, and Singer, 1996; Densmore & Donahue, 1995) and such publications make conceptually accurate, historical material far more available to educational researchers.

New Curricula Inspired by Original Historical Source Material

Piaget's theory of genetic epistemology compels educational researchers to examine the historical process of development within which scientific concepts were constructed. As interpreted by von Glasersfeld (1982, 1984) and Confrey (1994b) this theory rejects the notion that science is progressing linearly towards an increasingly accurate picture of "the way things really are." Scientific knowledge is viewed as actions and reflections on those actions. The theory calls for educational researchers to seek broad and diverse environments that "create the need" for an idea (Confrey, 1994a). It is here that historical studies can play a vital role, but the type of historical investigation that is required necessitates going beyond most readily obtainable secondary sources.

In order to gain useful historical insights that will direct researchers toward profound curricular innovations, one must examine carefully original historical documents that provide a full range of the ideas, tools, and environments that led to the formation of scientific concepts. Often, the most helpful ideas are those that have been pruned from both modern curricula and the usual histories of science (e.g. Allchin, in press). In order to use history as a source for the creation of innovative curricula one need not recreate exactly for students the historical problems and situations, nor does this process necessarily entail the inclusion of historical background.

Let us return to our example of analytic geometry to see how this process can work. Once one becomes aware of the fundamental difference between the analytic geometry of Descartes and that of Fermat, a question arises as to what kind of a curriculum might result from adopting some of the tools, actions, and concepts of the Cartesian approach. What happens if the mechanical or geometrical construction of curves is taken as a primary action, with coordinates and algebraic equations acting as secondary analytical facilitators? This view forces a reversal in the conceptual foreground and background. Students traditionally think of the coordinate system as the background, and then, by plotting pairs of numbers as points (usually from an equation), a curve is produced. Any geometrical analysis of the curve (such as a tangent line) occurs in the last foreground layer. Many students think that the only way to create a curve is to start with an equation, and, indeed, this view is reinforced strongly through the use of a graphing calculator.

Descartes' conceptual space reverses this entire process. A researcher might think first about what experiences of curve drawing remain in the curriculum. Generally, they are limited to a compass and perhaps a loop of string over two tacks used to draw ellipses. An in depth reading of Descartes' *Geometry* (1638) is then required, along with some modern intellectual history of the work's intentions (e.g., Lenoir, 1979). A reconceptualization occurs as one realizes that all algebraic curves can be drawn with linkages (i.e., simple mechanical devices consisting only of hinged rigid rods). Techniques for finding equations directly from the actions that produced a curve are scarcely known among modern mathematicians, although the most important technique is iterated similarity relations, which has been studied by Confrey as a fundamental cognitive issue in children's development of ratio concepts (1994a). History yields new curricular possibilities that link early cognition of ratio directly with more advanced scientific modeling concepts.

Further historical research into the history of curve drawing mechanisms and their role in the development of the notions of functions and calculus proves to be a rich topic. Such devices played very important roles in the work of Pascal, Newton, Leibniz, and others (Dennis, 1995; Dennis & Confrey, 1995; in press a). Hence, following a Piagetian constructivist model entails that the educational researcher then reflect on the curricular possibilities of curve drawing actions as a crucial developmental phase in the evolution of analytic geometry, mechanics, and calculus, although this does not mean that students must relive the exact historical chain of events.

The next questions involve how this historical environment with its tools, actions, and inquiries can be made fruitful for modern students. What are the appropriate modern environments? Clearly, the usual curriculum with its new attendant tool, the graphing calculator, will not work. Some of these devices can be built easily from cardboard, wood or strings, while the more complicated ones can be simulated readily using a computer with dynamic geometry software. This combination of physical reconstructions along with computer simulations allows students to engage and experiment actively with some of the most profound conceptions of seventeenth-century mechanics (Dennis, 1995; Dennis & Confrey, in press a).

Certainly, it must be asked what is to be gained intellectually from such a curricular innovation. Perhaps curve drawing was merely an awkward phase in mathematical history and contemporary students would do well to ignore it. A careful reading of historical sources along with a consideration of the modern possibilities leads to the opposite conclusion (Dennis, 1995). This reversed approach to analytic geometry involves students in direct modeling situations where the language of algebra is only as good as its ability to articulate what they can see happening in their own experiments (physically or on a computer). Student's investigations in historically-inspired, educational environments lead in directions that complement much of the latest educational theory (Dennis & Confrey, in press b). Curve drawing experiments conceivably could be done with young students well before the advent of algebra, and, in this environment, young students could begin discussing tangents, areas, and arc lengths not only before calculus but also well before algebra. This history suggests manipulatives that could provide effective background experiences so that the symbolic languages of algebra and calculus have strong physical referents.

An important methodological question is: what level of historical research is required to obtain insights that can lead to profound curricular innovations. In the example described above I began by reading a side-by-side French/English version of Descartes' *Geometry* (1638), followed by other works such as *The Mathematical Papers of Isaac Newton* (1967). Eventually I found that some of the best material is available only in original Latin texts from the seventeenth century, which can be found solely in rare books collections (e.g. Schooten, 1657).

Others who have pursued educational and historical research have had similar experiences. For example Reinhard Laubenbacher and David Pengelley at New Mexico State University recently have created college mathematics courses which are taught entirely from original historical sources. Their researches, for example, led them to some surprising conclusions that came from reading the original letters of Sophie Germain (1776-1831). They are now preparing a book entitled *Recovering Motivation in Mathematics: Teaching from Original Sources* which will make these sources accessible. In physics, Falk Riess, at Carl von Ossietzky University in Oldenburg, Germany has created a year of physics experiments done on replicas of original equipment for university students preparing to become secondary teachers. In order to recreate the environments that led to new concepts, Riess' research led him to many original historical archives and some unanticipated conclusions that would never have been found in secondary historical sources (Reiss, 1995; Heering, 1992). Such research is difficult, but, once done, vast rewards can be shared broadly.

Are historically grounded curricula are valuable? According to Falk Riess, having students verify Ohm's Law in electronics using a modern Ohmmeter is circular and absurd. Ohm's law is built into the device; it is assumed in the construction of the tool. He prefers his students to see the process by which Ohm came to formulate his famous law of electrical resistance. Most educational theorists agree that an understanding of the methods and conceptual frameworks of mathematics and science have increasingly become our educational goals. If we are to continue in this direction then the development of historically informed curricula will be crucial so that our students avoid such circularity.

The examples thus far described have all implied large curriculum changes involving many weeks of student activities, but historically informed curricula need not always imply such global conceptual innovations. Recently I worked on a project-based approach to an introductory course on discrete mathematics aimed mainly at students majoring in computer science. It was required that the students understand modular arithmetic notation and how it functions algebraically (i.e. $a = b \bmod n$, which means that *a* and *b* both have the same remainder upon division by *n*). The Professor teaching the course, Art Duval, wrote several introductory problems that he hoped would create the need for this notation, but, in every case he found direct ways to solve them without ever using modular notation. So what situation would create the need for modular arithmetic? Thinking about the question historically, the notation was created by C. F. Gauss in his work of 1801 the *Disquisitiones Arithmeticae* in which he developed his famous quadratic reciprocity theorems (Gauss, 1801). Since quadratic reciprocity and related theorems are considered beyond the scope of the course in question, a radical constructivist perspective would suggest that modular notation is being forced on students prematurely. As a compromise we created the following project:

> Given an arbitrary arithmetic sequence of integers with any starting point and any common difference, develop a method for determining whether such a sequence ever contains a perfect square. How far must you search in such a sequence to be sure that no perfect square will ever occur?

In the context of the course, this project is tied to a variety of other situations, but, even standing alone, it has the feel of computer science, and we both felt convinced that it was indeed the simplest investigation that might create the need for modular notation. Although no formal theorems on quadratic reciprocity are to be discussed, the project gets at the heart of quadratic reciprocity in that if students start checking examples randomly, roughly half of the sequences will contain squares. History was a great guide here in the search for a good, one-week project. This and related projects are now being tested at the University of Texas at El Paso as part of a National Science Foundation project.

Social History of Curricula and Implied Values

Mathematics and science curricula for kindergarten through grade 12 are controlled by a variety of state institutions from almost uniform, statewide mandates as in New York, Texas, or California to almost complete control at local school district levels in other states. University curricula rarely fall under such direct bureaucratic control; however, they often display remarkable uniformity nationally and even internationally (e.g., college calculus sequences). In this section, I examine how historical studies could inform educational research by showing how certain curricula came to be what they are, whose interests are being served by the use of these curricula, and what viable alternatives might exist. This approach to social history is situated in a largely Vygotskian perspective where science and mathematics are considered to be linguistic tools, and tools are seen as socially mediated agents that transform human endeavors in which society and the state have an interest (Confrey, 1994b).

Historical studies in this context can present researchers with an array of possible directions, and the most fruitful investigations may not always be in the same chronological frame of reference. For example, when thinking about the reform of school mathematics it is certainly necessary to look back at the last major attempt at reform: the "new math" movement of the early 1960s. What parts of this movement were successful? Why did other parts fail? Who created this curriculum and whose interests did it intend to serve? Why was the "new math" movement almost universally rejected by 1980? How did its rejection contribute to the curricular backlash of the 1980s known as the "back to basics" movement? Why did the "back to basics" movement fail even more spectacularly than the "new math" movement? It is crucial to consider these recent historical questions in the light of both culture and technology before proceeding to launch any kind of new research aimed at the reform of mathematics curricula, especially if such initiatives are to have large-scale government funding.

An understanding of such recent curricular history alone, however, is not enough to see clearly how certain concepts came to be standard curricula and what other alternatives are socially and culturally possible. When considering the example of mandatory mathematics curricula from a historical perspective the most immediately striking feature is that the majority of the curricula comes directly from seventeenth-century Europe. From the beginning of algebra in middle school to the end of differential equations in college, the curricula focus on mathematics that began with Descartes and Fermat about 1620 and concluded with Leonard Euler (1707-1783) about 1740. This includes all secondary and college mathematics for even the most capable students except for a tiny number who are majoring in mathematics and physics. The only important standard topics that fall outside this historical period are some Euclidean geometry from the third century B.C. and some statistics from the nineteenth century A.D.

Studies of these curricula must address larger historical issues than merely the past few decades of attempted reform. What are our social and cultural intentions? Why, for example, are mandatory mathematics curricula so firmly rooted in a narrow historical period? Most educational researchers are keenly aware that socio-cultural pedagogical reforms can never be made independently of curricula and so a larger historical perspective has to address both issues simultaneously. In order to illustrate the role of history here, I will return once again to the example that has been discussed in the previous two sections (analytic geometry) but, this time, the subject must be seen as part of a larger social history. What follows here is a very brief sketch of the kind of historical analysis that could provide profound insights for educational researchers. This analysis is provided as an example only of how one might begin to think through such issues, and it is not fully detailed or referenced.

There are some important social-historical reasons why most of our mandatory mathematics curriculum originates in seventeenth-century Europe, although these issues are not discussed in educational research usually. Most important, perhaps, is the fact that the formation of the modern state originates from that period. The two most important institutions that mark the origin of the modern state are a professional bureaucracy and a professional military. Government-supported scientific research programs and broadly based educational institutions appear only after the establishment of a modern state with these two institutions. Briefly put, the shift in power relations caused by the social transition to a modern state produced the "scientific revolution" (Foucault, 1977). During the early seventeenth century, these social changes took place very rapidly in France (Beik, 1985), and it is here that one finds the earliest development of algebra and analytic geometry, which remain parts of our modern curricula.

One must investigate how social and political environments influenced the development of mathematical conceptions. Consider again the work of Fermat and Descartes. Place the genesis of their ideas in a cultural setting and look at the differences between their approaches to mathematics in the light of their respective social positions.

Pierre Fermat was born into a moderately wealthy family in Languedoc. His father was a prosperous leather merchant and a "bourgeois second consul" in the region, a man whose fortunes were rising and who wanted to translate his finances into political power. The family chose a legal career for their son Pierre, as this was one common path for upward social mobility at that time. Pierre studied law at a new university, and, in anticipation of his graduation, his parents had purchased the offices of *Conseiller au Parlement de Toulouse* and *Commissaire aux Requêtes du Palais*. Pierre Fermat became Pierre de Fermat. He became a member of the new lesser nobility, the *noblesse de robe* (nobles of the robe). Throughout his life he would remain a member of the newly-organized, centralized bureaucracy of the emerging absolutist monarchy of

France. Fermat's mathematical investigations began in earnest once he took up his office (Mahoney, 1973).

Descartes, by contrast, was born into a family of the old nobility, the *noblesse de épeé* (nobles of the sword). He studied military geometry and law at the Jesuit school at La Fléche. As he was not the oldest son, he took up a military career and participated in several campaigns with the Dutch, Bavarian, and French armies. Having obtained enough of a fortune as a mercenary to live in modest comfort, Descartes spent the remainder of his life constructing a grand scheme for the creation of a new philosophical and scientific system in which mathematical abstraction played a key role (Lenoir, 1979; Katz, 1993).

Many aspects of their different approaches to analytic geometry can be explained by looking at the social backgrounds and intentions of these two men. Descartes' view always remains grounded in mechanics and engineering. He also was concerned with the relationship of his mathematics to the ancient geometrical and philosophical traditions, but the problems that he chose and the metaphors that he selected to describe them inevitably reveal his background in military engineering. For example he describes the phenomenon of refraction by describing a cannon ball breaking through a tightly stretched cloth. Fermat, by contrast, was a government bureaucrat. He began his career by making a series of arguments concerning tax collection and monetary policy. His legal arguments were mathematical and largely ignored because few people understood them. He wanted a way to display complicated numerical relationships. His mathematical treatises created the concept of graphs as a visual display of numerical phenomena, and it is not surprising in this context that he developed the first effective algorithms for solving maximum-minimum problems. Fermat's description of refraction thinks of light as information seeking the most efficient path.

It is revealing that in modern public schools we say that we are teaching analytic geometry in the Cartesian plane, but what actually is taught is almost entirely the method of Fermat. The coordinate system is always laid down first, and then used to make a picture of a numerical relationship (i.e., points are plotted on a grid from a preexisting equation). Students are rarely taught to construct curves geometrically and then choose an appropriate coordinate system to create an algebraic model of a geometrical action. It would seem that our educational intentions are aimed far more toward bureaucratic interests than engineering ones. One could go on to trace the social history of how these Cartesian mechanical curve drawing devices slowly disappeared from standard curricula only after many experiments convinced mathematicians that algebraic language, including calculus, was an adequate model of mechanics.

So now given the new social and technological environments, an educational researcher can ask how do we want to transform the study of analytic geometry. Dynamic geometry computer environments allow for the rapid geometric construction of curves (Dennis & Confrey, in press a). Descartes' approach can be made readily available now for students to explore (Dennis, 1995), but then researchers face the social question of whether society wants to institute a curriculum that has strong ties to mechanical engineering. Such a curricular shift would be greatly beneficial not only in engineering, but also in physics, astronomy, geology, and chemistry, where student's lack of experience with the physical models that underlie mathematical language is a constant drawback. On the other hand, perhaps the overriding interests of society lie more with data-oriented bureaucratic mathematics. The analytic geometry of Fermat is more appropriate in a computer spread-sheet environment and perhaps that is a dominant concern, but, if so, then an historical study of that conceptual framework would lead from Fermat to John Wallis (1616-1703), mathematician and code-breaker for Oliver Cromwell (Dennis & Confrey, 1996). Any balance of values that is to be achieved in curricula surely must benefit from a larger informed, social-historical view that addresses the question of how mathematics and science serve us. Everything old is new again, but socially different.

Conclusions

These three approaches to the use of history in mathematics and science education have different research implications but are not mutually exclusive with respect to development of curricula. For example, one might combine the first two methods by creating a set of historically based environments for students' investigations and then following up such investigations by having students look at various events that happened in history. Afterwards students could write essays that compare and contrast different historical investigations with each other and with what happened in their own class, thus achieving a richer constructivist curriculum along with a broader cultural interpretation.

Social historical investigations can shed important light on how to choose intellectual historical material that directs and informs the choices of curricula in accord with the social and philosophical intentions of a given educational research agenda. The process of original, scientific, historical research generally entails going back and forth between social and intellectual history; for example it is impossible to separate Descartes' geometry from his larger philosophical goals, and, in order to understand his mathematics, it is essential to see his philosophy in relation to his society (Lenoir, 1979). In educational research the historical and social setting of a concept must be compared constantly to current social settings in order to make appropriate choices of tools and environments.

Confrey's constructivist philosophy calls upon all teachers to be good listeners and to have the intellectual flexibility to respond to the voices of students with rich and stimulating activities. This entails that teachers must become, to some extent, impromptu curriculum developers in their classrooms. This can happen only if such teachers have access to a wealth of sound conceptual material, which should include good historical material that is tied to descriptions of possible learning environments that are conceptually rich. It is here that the fruits of educational research based on genetic epistemology ultimately must find their audience.

A narrow oversimplified history of science will not serve, but neither will impenetrable original sources. New works are appearing in this area that make these educational research goals increasingly obtainable. For example, Densmore and Donahue (1995) have published a translation of the central ideas from Newton's *Principia* along with commentaries that render this work understandable and yet preserves the essential geometry that makes it quite strange to a modern reader. Such resources make historically based educational research much easier, although the design of activities and environments remains a difficult task.

An initial step that would help to foster the kinds of discussions that need to take place would be to have educational researchers participate in seminars with historians of mathematics and science. Participating historians would need to be made aware of the needs and concerns of educational research. Although this might be difficult, such discussions often energize historical researchers when they see the profound impact that educational studies can have for students. I hope that such discussions will become a widespread part of both educational research and teacher education programs. For those who face the difficult task of creating and implementing rich and stimulating curricula in our schools, creative, well-directed, historical research can provide an abundant flow of diverse ideas.

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